Synthesis of Collinear Dipole Array of any Length with Minimum Standing Wave Ratio Using Particle Swarm Optimization

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Abstract – **In this paper, the authors propose a method based on particle swarm optimization for design of a non-uniformly placed collinear dipole array of any length. The array is designed to generate a pencil beam in the vertical plane with minimum standing wave ratio (SWR) and fixed side lobe level (SLL). A simultaneous optimization of excitation distribution and geometry (both inter element spacing and dipole length) of individual array elements is carried out to minimize side lobe level and standing wave ratio. Coupling effect between the elements is analyzed using induced EMF method and minimized in terms of SWR. Dynamic range ratio (DRR) of excitation distribution is fixed at an optimum value in order to obtain impedance matching condition. Phase distribution for all the elements is kept at zero degree. Numerical results show the effectiveness of the algorithm for the design of the array. Finally the performance of the PSO algorithm is compared with that of the differential evolution (DE) in order to present a comparative evaluation.**

Index Terms- **Particle Swarm Optimization (PSO), Collinear Dipole Array, Standing Wave Ratio (SWR), Induced EMF Method, Differential Evolution (DE).**

I. INTRODUCTION

The synthesis problem of an antenna array is related with the calculation of the excitation and geometric configuration that produce a desired pattern. Many methods have been adopted to achieve specified radiation pattern [1-12] for non-uniformly excited, non-uniformly spaced linear arrays. The analysis of non-uniformly spaced linear arrays was first proposed by Unz [1], who developed a matrix formulation to obtain the current distribution necessary to generate a desired radiation pattern. Array geometry was calculated either by thinning array elements selectively or positioning the array elements randomly along the desired direction.

Skolnik [2] employed dynamic programming for zeroing elements. Mailloux and Cohen [3] utilized the statistical thinning of arrays with quantized element weights to improve side lobe level performance. The Genetic Algorithm [4–6] and Simulated Annealing (SA) [7] were used to thin an array. Razavi and Forooragi [8] used pattern search algorithm for array thinning.

Later Harrington [9] developed an iterative method to reduce the sidelobe levels of uniformly excited N-elements linear arrays by employing unequal spacing. Literature described in [10-12] proposed different analytical methods for nonuniformly spaced array synthesis. In [10], the particle swarm optimization was applied for optimization of non-uniformly spaced antenna arrays and sidelobe level was reduced. In [11], with Neural Network (NN) and in [12] with least mean square, non-uniformly spaced arrays were synthesized. Most works consider the minimization of the sidelobe level at a fixed beam-width without considering mutual coupling (MC) effect. In a few recent works [13,14] driving point impedance matching has been derived with unequal spacing of elements.

In the proposed work we synthesize a non-uniformly spaced array consisting of radiators of unequal heights. In our work, particle swarm optimization (PSO) and DE is used for synthesis

of pencil beam pattern with specified SLL at a fixed beam-width and DRR value by optimizing inter-element spacing, antenna height and in-phase current amplitude. Coupling effect is minimized by minimizing standing wave ratio along with the dynamic ranges of excitation current amplitude. Impedance matrix is calculated using induced EMF method [16]. Active impedance is suitably varied according to the inter-element distances, antenna length and excitation distributions to become feed matched. At last we employ another evolutionary algorithm DE to compare the performances of both the algorithms to design the array.

II. FORMULATION

We consider a collinear array of 2N number of identical very thin dipole antennas, spaced at a distance d_n apart (center to center) along the Z-axis with its center at the origin. Excitation and geometry both are assumed to be symmetric with respect to the center of the array. Assuming sinusoidal current distribution of a very thin dipole antenna directed along Z-axis, the element pattern is as given by (1).

$$Elepat(\theta) = \frac{\cos\left(\frac{kl_n\cos\theta}{2}\right) - \cos\left(\frac{kl_n}{2}\right)}{\sin\theta} \qquad (1)$$

The far-field pattern $F(\theta)$ in the principal plane (*yz* plane) taking into account the element pattern with symmetric amplitude and phase distributions is represented by (2)

$$F(\theta) = \sum_{n=1}^{N} 2I_n \cos[kd_n\cos\theta] \times Elepat(\theta) \qquad (2)$$

Normalized power pattern in dB can be expressed as follows.

$$P(\theta) = 10\log_{10}\left[\frac{|F(\theta)|}{|F(\theta)|_{max}}\right]^2 = 20\log_{10}\left[\frac{|F(\theta)|}{|F(\theta)|_{max}}\right] \qquad (3)$$

Here *n* is the element number, $k = 2\pi/\lambda$ = free-space wave number, λ = wavelength at the design frequency, d_n is the inter-element spacing,

l_n is the length of *n*-th antenna element θ is the polar angle of far-field measured from broadside (0° to 180°), I_n = excitation current of *n*-th element,, $[V]$ the voltage matrix of size $N \times 1$ obtained from the expression $[V]_{N\times 1} = [Z]_{N\times N}[I]_{N\times 1}$ where $[Z]$ is the mutual impedance matrix of size $N \times N$. Where $Z_{n,m}$ is the mutual impedance between dipoles n and m [16]. Value of $Z_{n,m}$ depends on the geometry of the dipoles and their mutual geometric relations. PSO algorithm is used to optimize the antenna array shown in Fig.1. The radiation patterns (pencil beam) produced by the array is required to satisfy the condition of low SLL, impedance-matching and optimum dynamic range ratio. In order to optimize the antenna arrays according to the above three conditions, a
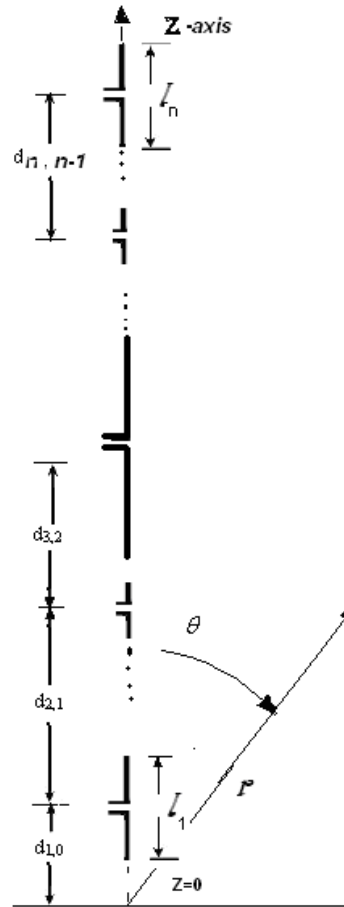

Fig.1 Collinear array of unequal dipoles

cost function J is formed with the weighted sum of three respective terms, as given by the following equation:

$$J = w_1 * (SLL - SLL_d)^2 + w_2 * SWR_{max} + w_3 * (DRR - DRR_d)^2$$
(4)

Where, SWR_{max} is the maximum SWR value. SLL, SLL_d, DRR, DRR_d are obtained and desired values of corresponding terms. DRR stands for dynamic range ratio that is computed from the given expression.

$$DRR = \max(I_n)/\min(I_n) \qquad (5)$$

Impedance matching is described in terms of SWR. According to the transmission line theory [10,11] input impedance $Z_{n,n}$ of each (n -th) element is defined as $Z_{n,n} = V_n/I_n$. Thus $Z_{n,n}$ has to be as close as possible to the characteristic impedance Z_0 (50Ω) of the transmission line that feeds the element for efficient radiation.

Impedance matching is obtained if $Z_{n,n} = Z_0$ i.e when SWR=1. Though for practical purpose maximum tolerable value of SWR is 2. The coefficients w_1, w_2 and w_3 are weight factors and they describe the importance of the corresponding terms that compose the cost function. PSO attempts to minimize the cost function to meet the desired pattern specification.

The radiation pattern on the vertical plane depends on the geometry of the array as well as on the excitation currents applied in the middle of the length of the dipoles. So we carried out simultaneous optimization of excitation and geometry of individual array elements to reduce SLL and SWR value. To generate desired pencil beam, length and spacing of each element is varied in the range 0.3 to 0.6 wavelengths and 0.6 to 1.2 wavelengths respectively. Excitation current phase is kept fixed at 0 degree. Excitation current amplitude is varied in the range 0 to 1. Excitation and geometry both are assumed symmetric about the center of the array. PSO is run for several iterations to optimize the collinear array.

III. OVERVIEW OF PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Eberhart and Kennedy in 1995 [17], inspired by social behavior patterns of organisms that live and interact within large groups. PSO algorithm starts with a group of random particles and then searches for optima by updating each generation. Each particle is treated as a volume-less particle in the n-dimensional search space. The i-th particle is represented as $X_i = (x_{i1}, x_{i2} \cdots, x_{in})$. At each generation, each particle is updated by the following two best values:

i) The first one is the best solution it has achieved so far. This value is called c-best value.

ii) Another is the best value obtained so far by any particle in the population. This best value is a global best and called g-best. When a particle takes part in the population as its topological neighbors, the best value is a local best and is called l-best.

In all iterations, these two best values are combined to adjust the velocity along each dimension, and that velocity is then used to compute a new move for the particle. The portion of the adjustment to the velocity influenced by the individual's previous best position (c-best) is considered the cognition component, and the portion influenced by the best in the neighborhood (l-best or g-best) is the social component.

With the addition of the inertia factor ω, by Shi and Eberhart [18] the updating equations of velocities and positions are as follows.

$$v_{i+1} = \omega * v_i + c_1 * random(0,1) * (cbest_i - x_i)$$
$$+ c_2 random(0,1) * (gbest - x_i) \qquad (7)$$

$$x_{i+1} = x_i + v_{i+1} \qquad (8)$$

Where random (0,1) is a random number independently generated within the range [0,1] and c_1 and c_2 are two learning factors, which control the influence of the social and cognitive components. In eqn. (7) if the sum on the right side exceeds a constant value, then the velocity in that dimension is assigned to $\pm V_{i\max}$. Thus, particle's velocities are clamped to the range of $\left[-V_{i\max},+V_{i\max}\right]$ which serves as a constraint to control the global exploration ability of PSO algorithm. This also reduces the likelihood of particles for leaving the search space.

Additional concepts about PSO and DE can be found in literature [17-21].

IV. RESULTS AND DISCUSSIONS

We consider a collinear array of 20 dipole antennas of radius 0.005λ . Array elements are randomly placed along z-axis. To generate a pencil beam, all excitation current phases are kept fixed at 0 degree. Excitation current amplitudes, inter element distances and antenna lengths all are varied in the range 0 to 1, 0.6 to 1.2 and 0.3 to 0.6 respectively. Desired DRR value of amplitude distribution is prefixed at 6.

Because of symmetry, only ten amplitudes, nine inter element distances and ten antenna heights are to be optimized using PSO. The algorithm is designed to generate a vector of 29 real values between zero and one. The first 10 values of the vector are mapped and scaled into desired amplitude weight (0 to 1) range and next nine values are mapped and scaled into desired intermediate spacing weight (0.6 to 1.2) range and last ten values are mapped and scaled to desired length weight (0.3 to 0.6) range. We consider inter element distances from center to center and place first element at a prefixed distance from the origin. For design specifications as given in Table 1 and Table 2, PSO is run for 200 iterations. In the numerical experiments, the parameters used by the PSO algorithm are: acceleration constant $c_1 = c_2 = 2$; time varying inertia weight ω that varies from 0.9 at the beginning to 0.4 toward the end of the optimization; maximum particle velocity is 1.0 and population size is 30.

Table 1 Desired and obtained result for the array using PSO

Design Parameters	Pencil Beam	
	Desired	Obtained
Side Lobe Level (dB)	-30.00	-28.03
Standing wave ratio (avg. SWR)	NA	2.01
Obtained DRR = 7.1		

Table 1 shows the desired and obtained results in absence of ground plane. There is a good agreement between the desired and synthesized results. For optimization purpose we consider 10 array elements and calculate the SWR values of individual elements. Maximum SWR value is minimized in each step. We also calculate the average SWR value and it is found to be 2.01. SLL value also meets the desired specification of SLL_d (-30dB). Desired dynamic range ratio of excitation distribution is fixed at 6.0.

Table 2 Current Amplitude, antenna height, inter element spacing and SWR for the array using PSO

n	Non-Uniformly spaced array			
	Current Amplitude I_n	Antenna Height	Spacing	SWR_n
±1	0.9254	0.54744	0.47372 (from origin)	2.7958
±2	0.7884	0.55128	0.9002	2.9125
±3	0.7929	0.54344	0.9176	2.7425
±4	0.7257	0.53736	0.88992	2.6501
±5	0.7268	0.49842	0.84949	1.6182
±6	0.4592	0.50192	0.83781	1.5267
±7	0.5785	0.46458	0.87301	1.1824
±8	0.4464	0.48906	0.90154	1.3211
±9	0.2422	0.50872	1.0278	1.5368
±10	0.1300	0.47268	0.7981	2.427

Parameters obtained from the algorithms are shown in Table 2. This table shows the length of the individual dipoles, inter-element distances and finally the SWRs at the feeding points of the dipoles. It must be noted that the 1st dipole is placed arbitrarily 0.47372 wavelength away from the origin (z=0). In broadside case, the excitation phases are not subject to optimization because the dipoles of broadside arrays are always in phase and thus the phase is kept at zero degree. DRR of the excitation is found to be 7.1. Because of symmetry, remaining ten elements are also to be excited with the same parameters. The advantage of this type of arrangement is the simplicity in feed network.

Radiation patterns using the optimized data are plotted below. Figure 2 shows the normalized absolute power patterns (pencil-beam) in dB using PSO for non-uniformly spaced collinear array elements.
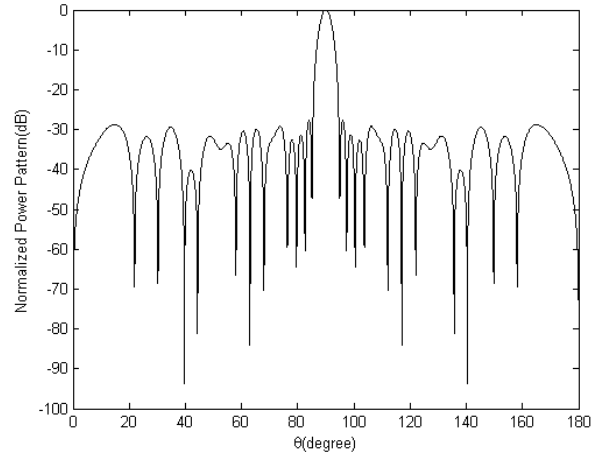

Fig. 2 Normalized absolute power patterns in dB using PSO

In order to evaluate the performance of the PSO, another optimization technique DE [19-21] is again employed to study the behavior of the proposed array. In DE, a trial solution for a parent is produced using the recombination of a mutant vector and the parent vector. The scaling factor [19] used in DE affects the differential variation between two solutions and was set to 0.8 in our experiments. Value of the crossover rate [19], which controls the change of the diversity of the population, was chosen 0.9. DE is run for 200 iterations and number of population is set at 30 in order to have similar computational time. Results obtained from DE are presented in Table 3 and Table 4. Radiation pattern using the DE optimized data is plotted in Fig.3.

Table 3 Desired and obtained result for the array using DE

Design Parameters	Pencil Beam	
	Desired	Obtained
Side Lobe Level (dB)	-30.00	-28.3684
Standing wave ratio (avg. SWR)	NA	1.8458
Obtained DRR = 5.7372		

Table 4 Current Amplitude, antenna height, inter element spacing and SWR for the array using DE

n	Non-Uniformly spaced array			
	Current Amplitude I_n	Antenna Height	Spacing	SWR_n
±1	0.9946	0.4045	0.4023 (from origin)	1.7194
±2	1.0000	0.5293	0.7065	2.2448
±3	0.9585	0.3716	0.7320	2.3100
±4	0.7762	0.4221	0.8130	1.2638
±5	0.6183	0.3973	0.6428	2.2075
±6	0.4918	0.3820	0.6077	1.7734
±7	0.5123	0.471	0.6554	1.1433
±8	0.3983	0.4504	0.7871	1.4095
±9	0.3564	0.5073	0.7527	1.8428
±10	0.1743	0.5031	0.9178	2.5432

Both the technique produces almost similar results. It is seen that, DE is more effective to find better solution.
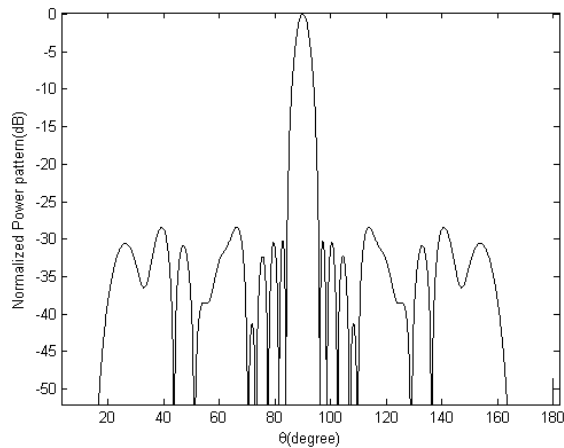

Fig. 3 Normalized absolute power patterns in dB using DE

The optimized results show good matching with desired specification in both the cases.
In our proposed design, phases are not required to generate pencil beam. In our work, the fitness function with fixed DRR value not only reduces coupling effect but also gives more freedom to optimization algorithm as it directs the algorithm where to stop. Beside this in article [14] antenna

heights are varied uniformly to make the element feed matched. However, in our work we varied height of each radiator individually which, makes the proposed technique more suitable for practical application. Kings [15] method is employed for evaluating the real and imaginary components of mutual impedance between any two radiators of arbitrary length. Moreover, in this paper we use another more recent evolutionary algorithm DE to optimize the array along with PSO to present a comprehensive overview between the two.

V. CONCLUSION

The use of particle swarm optimization in the synthesis of non-uniformly spaced collinear array of unequal lengths is presented. To design the collinear array an appropriate geometry and excitation is chosen in order to satisfy the desired criteria. Phase is prefixed at zero degree. It is seen that array performance is significantly enhanced by controlling the inter-element spacing. But unequal length of the antenna increases the coupling effect considerably. The proposed work is effective to determine the interference pattern due to parasitic wires or antennas adjacent to the fed antenna. Thus it enables us to approximate various practical problems in communication area. Though the application of formulas cited in the references remain limited for infinitely thin antennas only. The excitation and geometry both are symmetric in nature that greatly simplifies the feed network. Mutual impedance matrix is calculated using induced EMF method. In the proposed method, driving point impedance of each element is varied suitably by optimizing array geometry and excitation. Thus active impedances become matched with feed network and mutual coupling effect is compensated. Fixing the dynamic range ratio of excitation current amplitude to a lower value with little compromise on the design specifications further reduces the effect of coupling. There is a good agreement between desired and obtained results using PSO. Another evolutionary algorithm DE is also employed to present a comparative evaluation of two algorithms. Simulation results show that DE is better than PSO in terms of SLL, DRR and SWR. We can extend this work further by using method

of moments. The technique is capable of optimizing more complex geometries and therefore is suitable for many applications in the field of communication.

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