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Gray and Dark Spatial Solitary Waves In Left-Handed Waveguide Structure

H. M. Mousa[1] and M. M. Shabat[2]

[1]Physics Department, Al Azhar University, Gaza, Palestinian Authority
e-mail: H.mousa@ alazhar. edu.ps

[2]Physics Department, Islamic University, Gaza , P.O.Box 108, Palestinian Authority,
e-mail: shabat@ mail.iugaza. edu.ps

Abstract - **The propagation characteristics of both TE gray and dark solitary waves in a waveguide structure consisting of left handed material LH film sandwiched in a nonlinear defocusing medium is investigated. In (LH) film both permittivity and magnetic permeability are negative in definite frequency range. We study dispersion and grayness properties of the solitary waves. We found that the implementation of the left handed material stimulate the backward traveling of the waves with high intensity at the film boundaries. We also found that higher values of wave's grayness are obtained for relatively small magnetic permeability of LH film. These results may be used in designing microwave-photonic devices which have found increasing use in information and telecommunication technologies,**

Index Terms- **Dark waves, dispersion relation, grayness, left handed material, nonlinear medium.**

I. INTRODUCTION

Recently, the propagation of a light wave in uniform nonlinear media has been a subject of great interest because of its potential applications in optical switching, signal processing and communications [1]. Self-guidance of a light wave is based on the nonlinearity of a medium. The nonlinearity of materials consists of two varieties, self –focusing and self-defocusing. Self –focusing nonlinear medium can trap a gray solitary wave [2], whereas a self-defocusing medium could support a dark solitary wave [3] which has null intensity at the center. In the absence of nonlinearity, a nonguiding structure at low power may be converted to a guiding structure at high power when nonlinearity takes effect. Chen et. al. [4] examined the propagation of light waves in a thin linear film bounded by self –defocusing nonlinear media. Both gray and dark solitary waves are found to be trapped –mode patterns which contrast with self-defocusing medium in which only dark spatial solitary waves are guided. Within the last several years it was realized that left handed material (LHM) or metamaterial possess unusual properties which include resonant enhancement of evanescent fields, potentially enabling near-perfect imaging below the diffraction limit and leading to a new class of optical devices [5,6]. These LHM grasped great attention of many researchers' worldwide. Interest is focused on the propagation of electromagnetic waves in artificial materials, and particularly on materials with negative index of refraction: materials which are designed to exhibit both negative permeability and permittivity over predetermined range of frequencies. Ustinov et. al.[7] have experimentally observed and studied the generation of microwave spin-electromagnetic wave in the form of bright, gray, or black spin-wave solitons and chaos based on ferrite-ferroelectric metamaterial structure. Dynamic chaos offers large information capacity and security of communication. Mandal et al[8], designed metamaterials into an antenna with split-ring-resonator (SRR). a Klein-Gordon equation was derived which gave rise to both dark and bright solitons that showed interesting behavior against non dimensional time. Mousa and Shabat [9,10] have examined the propagation characteristics of nonlinear transverse electric (TE) surface waves in a left-handed material and magnetic superlattices (LANS) waveguide structures. In this paper, we investigate the propagation characteristics of both TE gray and dark solitary waves guided by a structure. This structure consists of a LHM film and is sandwiched between nonlinear cover and substrate.

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II. DERIVATION OF THE DISPERSION RELATION

The propagation of TE waves through a thin linear planar core of left handed material (LHM) with thickness 2d surrounded by a self-defocusing cladding and substrate is considered. The geometry structure of the problem considered here is shown in Fig.(1). The coordinate origin $(x=0)$ is the center of the core. LHM film occupies the region $|x| \prec d$. The nonlinear media occupy the region $d \prec x \prec -d$.

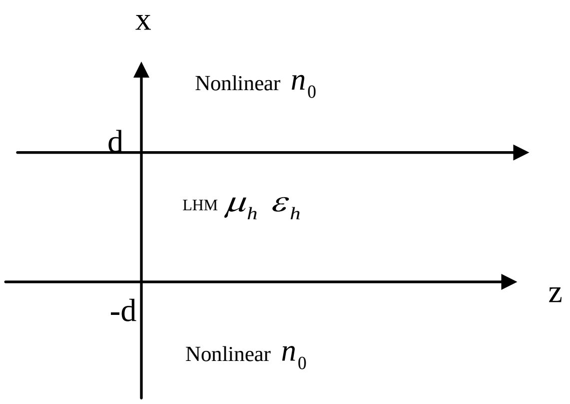

Fig.1. Spatial solitary waves waveguide composed of LHM film and nonlinear media.

We present the dispersion equation for transverse electric (TE) waves propagating in the z direction with a propagation wave constant in the form $\exp[i(k_z z - 2\pi ft)]$, f is the operating frequency. The electric and magnetic field vectors for TE waves propagating along z-axis with angular frequency ω and wave number k_z are defined as:

$$\begin{aligned} E &= \left[0\,,\, E_y(\omega, z),\, 0\,\right]\exp\, i\left(k_z\ z-\omega\, t\right) \\ H &= \left[H_x(\omega\,, z)\,, 0, H_z(\omega\,, z)\right]\exp\, i\left(k_z\ z-\omega t\right) \end{aligned} \tag{1}$$

The wave equation in each media is obtained from Maxwell's equations :

A- In LH film

Both a negative dielectric permittivity and permeability are written as [9-11]:

$$\varepsilon_h(\omega) = 1 - \frac{\omega_p^2}{\omega^2},\ \mu_h(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \tag{2}$$

with plasma frequency ω_p and resonance frequency ω_0 .

The wave equation can be found easily from the Maxwell's equations as :

$$\frac{\partial^2 E_y}{\partial x^2} - k^2 \nu^2 E_y = 0 \tag{3}$$

With $k\nu = k\sqrt{n_z^2 - n_h^2}$ is decay constant of the waves in LH film, $n_h = \sqrt{\mu_h \varepsilon_h}$ is refractive index of LH film and the effective wave index is $n_z = k_z / k$, k is the wave propagation length in free space. The symmetric(even) solution of Eq (3) corresponding to the gray waves has the form:

$$E_y(y) = A e^{ik_z z} \cosh[\nu k x],$$

$$H_z = (1/i\omega\mu_0\mu_h)\frac{\partial E_y^l}{\partial x} \tag{4a}$$

The symmetric(odd) solution of Eq(3) corresponding to the dark waves has the form

$$E_y(y) = A e^{ik_z z} \sinh[\,\nu k x],\ |x| \prec d \tag{4b}$$

A is an amplitude coefficient which can be determined by the boundary conditions.

B- In nonlinear medium

The Maxwell's equations can be written as:

$$\begin{aligned} \nabla \times \overline{E} &= i\,\omega\,\mu_o \mu^{nl}\,\overline{H} \\ \nabla \times \overline{H} &= -i\omega\ \varepsilon_s \varepsilon^{nl}\,\overline{H} \end{aligned} \tag{5}$$

The wave equation is:

$$\frac{\partial^2 E_y}{\partial x^2} + \left(k^2\ \varepsilon^{nl}\mu^{nl} - k_z^2\right)E_y = 0 \tag{6}$$

Where, $n = \sqrt{\mu^{nl}\varepsilon^{ni}}$ is the refractive index of the nonlinear defocusing medium. It is considered to be as: $n^2 = n_0^2 - n_2|E|^2$.

The linear part of refractive index is n_0 and n_2 is the nonlinear coefficient. The wave equation becomes:

$$\frac{\partial^2 E_y}{\partial x^2} + k^2\gamma^2\ E_y - k^2 n_2 E_y^3 = 0 \tag{7}$$

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With $k\gamma = k\sqrt{n_0^2 - n_z^2}$ is the decay constant of the waves in the nonlinear medium.
The symmetric(even) solution for Eq.(7) is given by [4]

$$E_y(y) = e^{ik_z z}\begin{cases} -\left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(x+x_1)/\sqrt{2}\right], & x \prec -d \\ \left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(x-x_1)/\sqrt{2}\right], & x \succ d \end{cases} \quad (8a)$$

The symmetric (odd) solution for Eq.(7) is given by

$$E_y(y) = e^{ik_z z}\begin{cases} \left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(x+x_1)/\sqrt{2}\right], & x \prec -d \\ \left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(x-x_1)/\sqrt{2}\right], & x \succ d \end{cases} \quad (8b)$$

For gray waves (even solution) The continuity of E_y and H_z at the boundaries $x = -d$ and at $x = d$ leads to the following equations

$$-\left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(-d+x_1)/\sqrt{2}\right] = A\cosh[\nu k d] \quad (9a)$$

$$\left(\gamma/\sqrt{n_2}\right)\tan ch\left[\gamma k(d-x_1)/\sqrt{2}\right] = A\cosh[\nu k d] \quad (9b)$$

$$\left(\gamma^2/\sqrt{2n_2}\right)\sec h^2\left[\gamma k(-d+x_1)/\sqrt{2}\right] = (\nu A/\mu_h)\sinh[\nu k d] \quad (9c)$$

$$\left(\gamma^2/\sqrt{2n_2}\right)\sec h^2\left[\gamma k\ (d-x_1)/\sqrt{2}\right] = (\nu A/\mu_h)\sinh[\nu k d] \quad (9d)$$

By dividing Eq.(9c) by Eq.(9a) or Eq.(9d) by Eq.(9b), the dispersion equation of the waves is then obtained as:

$$\frac{\gamma\mu_h}{\nu\sqrt{2}} = \frac{\tan ch\left[\gamma k\ (d-x_1)/\sqrt{2}\right]\tan ch(\nu k\ d)}{\sec h^2\left[\gamma k(d-x_1)/\sqrt{2}\right]} \quad (10a)$$

Suppose that $u = \left[\gamma k\ (d-x_1)/\sqrt{2}\right]$ (10b)

And $Q_h = \tan ch\, u = \dfrac{e^{2u}-1}{e^{2u}+1}$, then

$$2u = \ln\left(\frac{\tan ch u + 1}{1 - \tan ch\, u}\right) \quad (10c)$$

By substituting Eq.(10b) into Eq.(10c) leads to the following

$$x_1 = \frac{1}{\gamma k\sqrt{2}}\ln\left(\frac{1-Q_h}{1+Q_h}\right) + d\,, A = \frac{Q_h\gamma}{\sqrt{n_2}\cosh\ (\nu k d)} \quad (11a)$$

By substituting Eq.(10b) and Eq.(10c) into Eq.(10a) leads to the following

$$Q_h^2 + \frac{\nu\sqrt{2}}{\gamma\mu_h}Q_h\tan ch(\nu\, k\, d) - 1 = 0 \quad (11b)$$

The solution of Eq.(11b) gives:

$$Q_h = \frac{-\nu}{\gamma\mu_h\sqrt{2}}\tan ch(\nu k d) + \left(\frac{\nu^2}{2\gamma^2\mu_h^2}\tan ch^2(\nu k d) + 1\right)^{1/2}, \quad (12)$$

The electric field profile of the gray waves is :

$$\bar{E}_y = \frac{\sqrt{n_2}}{\gamma}E_y e^{-ik_z z} = \begin{bmatrix} -\tan ch\left[\gamma k(x+x_1)/\sqrt{2}\right], & x \prec -d \\ Q_h\dfrac{\cosh[\nu k x]}{\cosh[\nu k d]}, & |x| \prec d \\ \tan ch\left[\gamma k(x+x_1)/\sqrt{2}\right], & x \succ d \end{bmatrix} \quad (13)$$

The waves intensity is $\left(\left|\bar{E}_y\right|^2\right)$ and the grayness (G) of the gray solitary waves is measured [4] by $\min\left(\left|\bar{E}_y\right|^2\right) = A^2 n_2/\gamma^2$. It depends on the structure parameters and the value of n_z ranges from n_h to n_0 for its existence. It is defined as:

$$G = \frac{Q_h^2}{\cosh^2(\nu k d)} \quad (14)$$

In similar way, for dark waves(odd solution), the dispersion equation is

$$\frac{\gamma\mu_h}{\nu\sqrt{2}} = \frac{\tan ch\left[\gamma k\ (d-x_1)/\sqrt{2}\right]}{\tan ch(\nu k\ d)\sec h^2\left[\gamma k(d-x_1)/\sqrt{2}\right]} \quad (15)$$

With $A = \dfrac{\gamma Q_h}{\sqrt{n_2}\sinh\ (\nu k d)}$,

$$Q_h = \frac{-\nu}{\sqrt{2}\gamma\mu_h\tan ch(\nu k d)} + \left(\frac{\nu^2}{2\gamma^2\mu_h^2\tan ch^2(\nu k d)} + 1\right)^{1/2} \quad (16)$$

III. NUMERICAL RESULTS AND DISCUSSION

In the present work, the numerical calculations for a LH film , are taken with the following parameters :
$\omega_p/2\pi = 10GHz,\ \omega_0/2\pi = 4GHz,\ and\ F = 0.56$ [9-11]. When $n_0 \succ n_z \succ n_h$, the dispersion equation (10a) has been solved to compute the frequency versus the effective wave number for different values of the LH film thickness d. Fig.2 shows that for this set of parameters, the frequency range in which both ε_h and μ_h are negative is from 4GHz to 6GHz. In this range, the computed dispersion curves (f versus n_z) for different values of LH film thickness show that the gray waves propagate in the backward

wave direction. We notice that both the wave phase velocity $v_p = \frac{\omega}{k_z}$ and the wave group velocity $v_g = \frac{\partial\omega}{\partial k_z}$ dispersions are affected by the thickness of the LH film(d) where v_p decreases to positive values and v_g decreases to negative values by decreasing the LH thickness d. The curves are shifted to higher negative values of effective index n_z which realizes long propagation length and best guidance of the waves. Fig.3 displays the electric field and the intensity profiles of gray (even) solitary waves . We see that both the electric field and the intensity of the waves has the higher value at the film boundary $(x = \pm d)$ where $\bar{E}$ =1.6 and the intensity $|\bar{E}|^2$ =2.6. At $x = \pm 6d$, i.e. in nonlinear medium, $\bar{E}$ and $|\bar{E}|^2$ values decrease to 1. Fig.(4a,b) shows the electric field and the intensity profiles of dark (odd) solitary waves . We see that both the electric field and the intensity of the waves have the higher value at the film boundary $(x = \pm d)$ where $\bar{E}$ =2.2 and the intensity $|\bar{E}|^2$ =5. At $x = 0$, the electric field profile has null intensity. At $x = \pm 6d$, i.e. in nonlinear medium, $\bar{E}$ and $|\bar{E}|^2$ values decrease to 1. This intensity sharpness for both gray and dark waves is effect of the implementation of LH film. This result is different from that obtained by Chen et. al [4] for right handed film (RH). Fig.5 demonstrates the grayness of gray solitary waves G versus (n_z / n_0) for different values of film thickness ($\frac{n_h d}{\lambda} = 0.5$, 0.6, 0.7, 0.8). Using Eq. (12,14) at $v = 0$ $(n_z = n_h)$ or d =0, G equals unit. It decreases as $(n_z \prec n_0)$ and then increases sharply as $(n_z = n_0)$. For curve (1) at $\frac{n_h d}{\lambda} = 0.5$, G increases to the value of (2.5) . This result is different from that predicted by [4] for RH film in which G decreases to zero when $n_z = n_0$. The effect of permeability of LHM (μ_h) on the waves grayness is illustrated by Fig.(6). As μ_h decreases to the values $(-1, -0.5, -0.35)$, G increases to the values of (2.5, 10, 20) at $n_z = n_0$. This means that lower negative values of μ_h realizes higher grayness of TE waves. Fig.7 describes variation of G as a function of film thickness $\frac{n_h d}{\lambda}$ for a series values of n_z / n_0. It displays at $\frac{n_h d}{\lambda} = 0.2$, G is increased to the values of (1.3, 1.7, 2.6) by increasing the effective index n_z / n_0 to the values of (0.85, 0.9, 0.95).

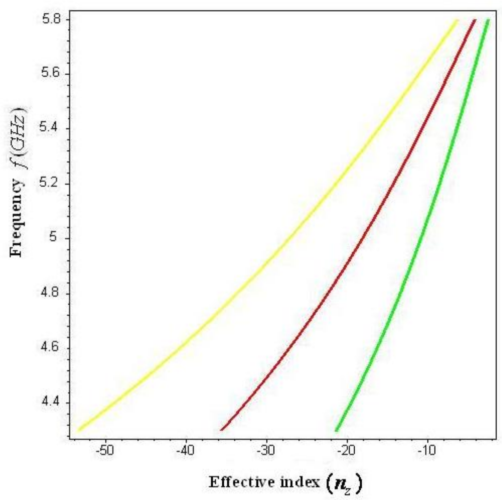

Fig.2. Dispersion curves of gray solitary waves for, $d = 100\mu m$ (yellow), $d = 150\mu m$ (red) and $d = 250\mu m$ (green). The curves are labeled with, $\mu_h \prec 0$, $n_h / n_0 = 0.8$, and $n_z / n_0 = 0.9$.

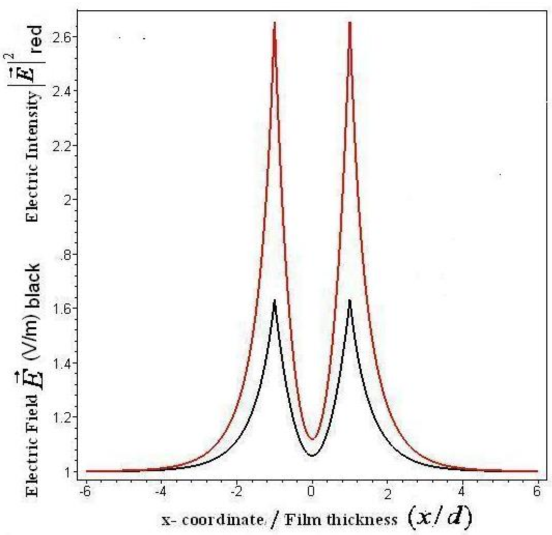

Fig.3. Demonstration of the field $\bar{E}$ (black) and the intensity $|\bar{E}|^2$ (red) profiles of the gray solitary waves. The curves are labeled with $\mu_h = -1$, $n_h / n_0 = 0.8$, $n_z / n_0 = 0.9$, and $k\nu d = 1$.

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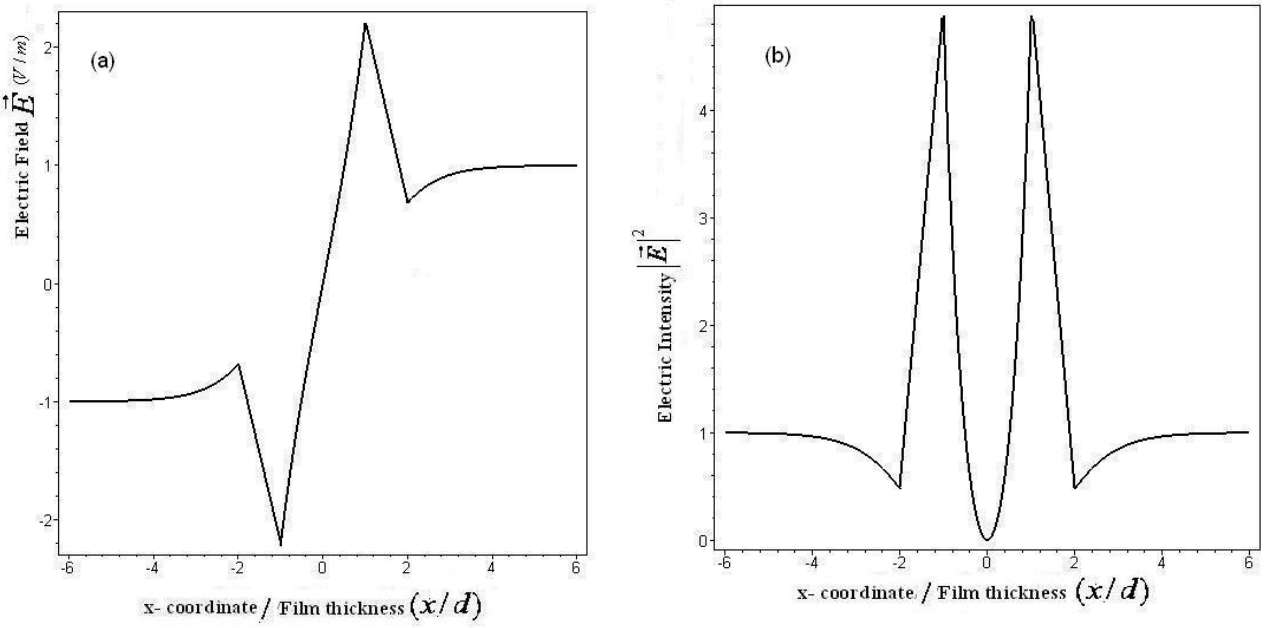

Fig.4. (a)Demonstration of the field $\bar{E}$ and (b) intensity $|\bar{E}|^2$ profiles of the dark solitary waves. The curves are labeled with $\mu_h = -1,\ n_h/n_0 = 0.8$, $n_z/n_0 = 0.9$ and $k\nu d = 1$.

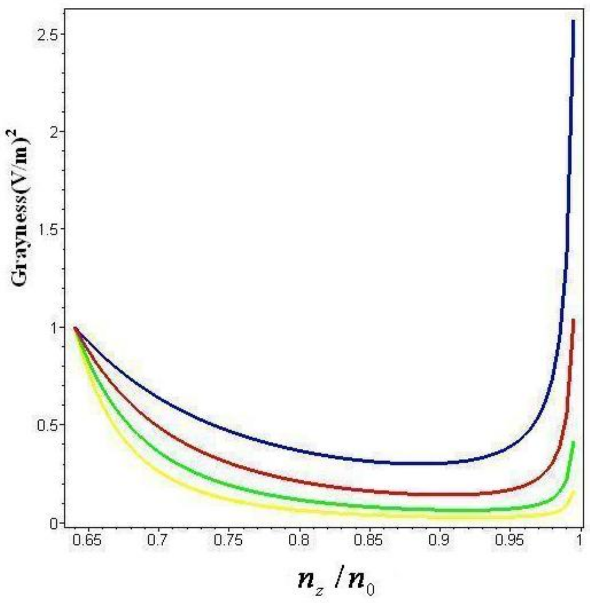

Fig.5. The grayness of gray solitary waves versus the wave index n_z for fixed film widths(1) $\frac{n_h d}{\lambda} = 0.5$ (blue), $\frac{n_h d}{\lambda} = 0.6$ (red), $\frac{n_h d}{\lambda} = 0.7$ (green), $\frac{n_h d}{\lambda} = 0.8$ (yellow), $\mu_h = -1,\ n_h/n_0 = 0.8$.

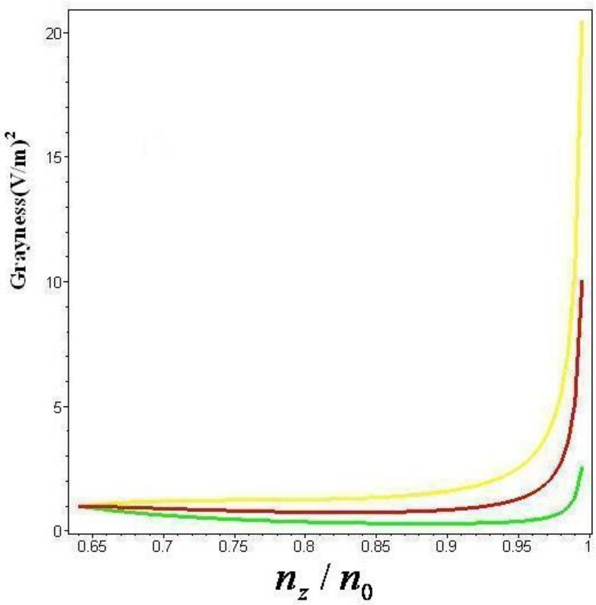

Fig.6. The grayness of gray solitary waves versus the wave index n_z for $\mu_h = -0.35$ (yellow), $\mu_h = -0.5$ (red) and $\mu_h = -1$(green), $\frac{n_h d}{\lambda} = 0.5$, $n_h/n_0 = 0.8$.

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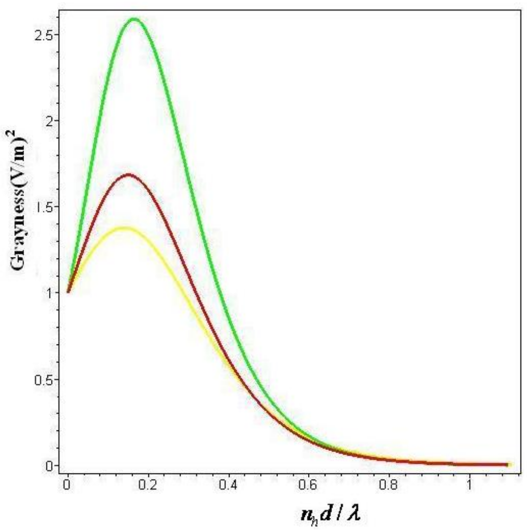

Fig.7. The grayness of gray solitary waves versus the film width for wave index $n_z/n_0 = 0.95$ (green), $n_z/n_0 = 0.9$ (red) and $n_z/n_0 = 0.85$ (yellow). $\mu_h = -1$ and $n_h/n_0 = 0.8$

VI.CONCLUSIONS

We investigated the propagation characteristics of both gray and dark solitary TE waves in a thin LH film bounded by self-defocusing nonlinear-media. Both gray and dark solitary waves are found to be possible trapped-mode patterns. The implementation of LH film heightens the waves intensity at the film boundary. Moreover, lower negative values of magnetic permeability of LH film realize higher grayness of TE waves. The grayness is also increased to high values when the effective wave index equal to the refractive index of the nonlinear medium. This contrasts with RH film in which the grayness drops to zero. The obtained results show many interesting features which may be used in designing future microwave-photonic devices.

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