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A Rigorous Study of Complex Modes by Applying the Transverse Operator in a Rectangular Guide of Ferrites

H. Sakli*[1], H. Benzina[2], T. Aguili[1], J. W. Tao[3]

[1]Ecole Nationale d'Ingénieurs de Tunis (Labo SYS'COM), Le Belvédère, 1002 Tunis, Tunisia
*e-mail : saklihedi@yahoo.fr ;Tel : (216) 97 904 982
e-mail : taoufik.aguili@enit.enu.tn
[2]Ecole Nationale d'Ingénieurs de Gabès, Rue Omar Ibn El Khattab, 6029 Gabès, Tunisia
e-mail : hafed.benzina@ieee.org
[3]Groupe de Recherche en Electromagnétisme (GRE), ENSEEIHT-INPT, 31071 Toulouse, France
e-mail : tao@enseeiht.fr

Abstract- **In hyper-frequencies, the ferrites are characterized by a tonsorial permeability which represents their inducted anisotropy under a magnetic field. This specific property makes the good formulation of the "Maxwell equation" harder.**

In this paper, a rigorous study of the formulation of the transverse operation method (TOM) is presented by a homogeneous rectangular structure of ferrites with transverse anisotropy which will be followed by the application of the Galerkin Method.

Our study essentially focuses on the determination of the propagation constants and the electromagnetic fields in a rectangular wave-guide of ferrites magnetized longitudinally by a static magnetic field. Hence, we'll show the existence of complex modes in the ferrite structure.

Index Terms- **anisotropic medium, complex modes, ferrite, TOM.**

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I. INTRODUCTION

The studies of the electromagnetic waves propagation in the guides charged by ferrites are present in many applications in the microwave circuits like the phase-converter, circulators, the isolators and the filters.

Despite the abundance of empirical studies in this domain, we can deduce a lack of theoretical contributions almost related to the propagation of higher order modes in the guides totally full of magnetized ferrites in the longitudinal direction.

The transverse operator which is another formulation of the Maxwell equations is used here to define the rectangular waveguide characteristics charged of ferrites. This method consists in eliminating the longitudinal components and resolving the propagation equation by developing the transverse fields in series of modes of a closed structure (empty metal guide)

The elimination of the longitudinal fields in the Maxwell equations paves the way for an operator L, named the transverse operator, to appear. The resolution of the propagation equation by the Galerkin method leads to an eigenvalues equations.

The permeability of the ferrites magnetized longitudinally is expressed by the tensor of Polder.

$$\overline{\overline{\mu}} = \mu_0 . \begin{bmatrix} \mu & -j\kappa & O \\ j\kappa & \mu & O \\ 0 & 0 & \mu_{rz} \end{bmatrix} = \overline{\overline{\mu}}_{rf} . \mu_0 \qquad (1)$$

Where μ, κ and μ_z are real quantities.

For a partial magnetization of the ferrites, Green [1] and Shloeman [2] give the empirical expressions of μ, κ and μ_z:

$$\mu_d = \frac{1}{3}\left\{1 + 2\sqrt{1 - \left(\frac{\omega_m}{\omega}\right)^2}\right\} \qquad (2)$$

Where: $\omega_m = \gamma(4\pi M_s)$

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ω is the pulsation, γ is the gyromagnetic constant $(\gamma = 2.8\,MHz/Oe)$ and $4\pi M_s$ is the saturation magnetization.

$$\mu = \mu_d + (1-\mu_d)(4\pi M / 4\pi M_s)^{3/2} \qquad (3)$$

$$\kappa = \frac{\gamma(4\pi M)}{\omega} \qquad (4)$$

$$\mu_{rz} = \mu_d \cdot \left(1 - \frac{4\pi.M}{4\pi.M_S}\right)^{5/2} \qquad (5)$$

Where $4\pi M$ is the magnetization.

II. FORMALISM OF THE TRANSVERSE OPERATOR

Given a system of rectangular coordinates, we consider a structure which comprises media characterised by their relative tensorial permittivity and permeability.

$$\overline{\overline{\varepsilon}}_r = \begin{bmatrix} \overline{\overline{\varepsilon}}_t & \varepsilon_{tz} \\ \varepsilon_{zt} & \varepsilon_{zz} \end{bmatrix} \;;\; \overline{\overline{\mu}}_r = \begin{bmatrix} \overline{\overline{\mu}}_t & \mu_{tz} \\ \mu_{zt} & \mu_{zz} \end{bmatrix} \qquad (6)$$

The Maxwell equations are:

$$rot\,\vec{E} = -j\omega\mu_0 \overline{\overline{\mu}}_r . \vec{H} \qquad (7)$$

$$rot\,\vec{H} = j\omega\varepsilon_0 \overline{\overline{\varepsilon}}_r . \vec{E} \qquad (8)$$

By eliminating the longitudinal component of the electromagnetic fields, (7) and (8) become [3]:

$$\hat{L}\Phi = j\eta\partial_z\Phi \qquad (9)$$

$\hat{L}$ is the transverse operator defined by :

$$\hat{L} = \begin{bmatrix} \hat{L}_{11} & \hat{L}_{12} \\ \hat{L}_{21} & \hat{L}_{22} \end{bmatrix} ,$$

with :

$$\hat{L}_{11} = k_0 \overline{\overline{\varepsilon}}_E - 1/k_0 \partial_t [1/\mu_{zz} \partial_t^+] \qquad (10)$$

$$\hat{L}_{12} = -\varepsilon_{tz}/\varepsilon_{zz}\partial_t^+ - \partial_t \mu_{zt}/\mu_{zz} \qquad (11)$$

$$\hat{L}_{21} = -\mu_{tz}/\mu_{zz}\partial_t^+ - \partial_t \varepsilon_{zt}/\varepsilon_{zz} \qquad (12)$$

$$\hat{L}_{22} = k_0 \overline{\overline{\mu}}_E - 1/k_0 \partial_t [1/\varepsilon_{zz} \partial_t^+] \qquad (13)$$

We have

$$\overline{\overline{\varepsilon}}_E = \overline{\overline{\varepsilon}}_t - \varepsilon_{tz}\varepsilon_{zt}/\varepsilon_{zz} \;,\; \overline{\overline{\mu}}_E = \overline{\overline{\mu}}_t - \mu_{tz}\mu_{zt}/\mu_{zz}$$

$$\partial_t^+ = [-\partial_y \quad \partial_x] \,,\; \partial_t = \begin{bmatrix} \partial_y \\ -\partial_x \end{bmatrix},\; \eta = \begin{bmatrix} 0 & \eta_0 \\ \eta_0 & 0 \end{bmatrix},$$

$$\eta_0 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, k_0 = \omega\sqrt{\varepsilon_0\mu_0} \,,\; \Phi = [E_t \quad H_t^{'}]^t$$

with $$H^{'} = j\sqrt{\mu_0/\varepsilon_0}.H \qquad (13a)$$

The longitudinal components are related to the transverse fields by:

$$H_z^{'} = -\mu_{zt}H_t^{'}/\mu_{zz} + \partial_t^+ E_t/\mu_{zz}/k_0 \qquad (14)$$

$$E_z = -\varepsilon_{zt}E_t/\varepsilon_{zz} + \partial_t^+ H_t^{'}/\varepsilon_{zz}/k_0 \qquad (15)$$

The so obtained relations are valuable to resolve the problem in a guide which contains inhomogeneous media, isotropics or anisotropics and dissipatives.

III. TRANSVERSE OPERATOR IN A GUIDE OF LONGITUDINALLY MAGNETIZED FERRITE

In a ferrite magnetized longitudinally and without loss, the relative permittivity is scalar (tensor of diagonal permittivity), and the relative permeability is tensorial and is given by equation (1). These conditions lead to $\hat{L}_{12} = \hat{L}_{21} = 0$.
So, the equation (9) becomes:

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$$\partial_z E_t = -j\eta_0 \hat{L}_{22} H'_t \qquad (16)$$

$$\partial_z H'_t = -j\eta_0 \hat{L}_{11} E_t \qquad (17)$$

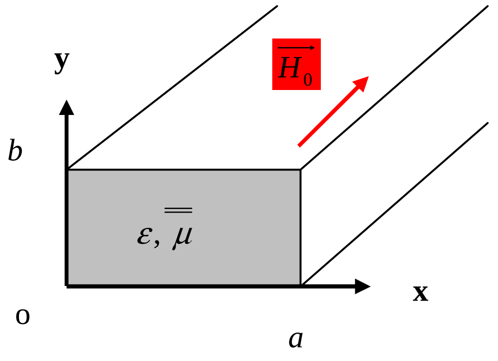

Fig.1. Geometry of the rectangular guide totally full of ferrite

By considering the propagation in the direction Oz, we have:

$$\Phi(x, y, z) = \Phi(x, y).\exp(-jk_z z) \qquad (18)$$

We obtain, then, the following fundamental equation of (16) and (17)

$$\hat{L}\Phi = k_z \eta \Phi \qquad (19)$$

Because of $\hat{L}$ is independent of z when we derive the system (16) and (17) with respect to z we obtain two decoupled equations in E_t and H'_t. The equation (19) becomes then [3]:

$$\hat{L}'\Phi = k_z^2 \eta' \Phi \qquad (20)$$

With

$$\hat{L}' = \begin{bmatrix} \hat{L}'_{11} & 0 \\ 0 & \hat{L}'_{22} \end{bmatrix}, \text{ and } \eta' = \begin{bmatrix} \eta_0 & 0 \\ 0 & \eta_0 \end{bmatrix}. \qquad (20a)$$

In these expressions:

$$\hat{L}'_{22} = \hat{L}_{11}\,\eta_0\,\hat{L}_{22} \qquad (21)$$

The decoupled equation in $\vec{E}_t$ becomes then:

$$\boxed{\hat{L}'' E_t = k^2 E_t} \qquad (22)$$

Or in another way :

$$\begin{cases} \hat{L}''_{11} E_x + \hat{L}''_{12} E_y = k^2 E_x & (23) \\ \hat{L}''_{21} E_x + \hat{L}''_{22} E_y = k^2 E_y & (24) \end{cases}$$

With:

$$\hat{L}''_{11} = \partial_x^2 + \frac{\mu}{\mu_z}\partial_y^2 + k_0^2 \varepsilon\mu + j\frac{\kappa}{\mu_z}\partial_x\partial_y \qquad (24a)$$

$$\hat{L}''_{12} = (1 - \frac{\mu}{\mu_z})\partial_x\partial_y - j[k_0^2 \varepsilon\kappa + \frac{\kappa}{\mu_z}\partial_x^2] \qquad (24b)$$

$$\hat{L}''_{21} = (1 - \frac{\mu}{\mu_z})\partial_x\partial_y + j[k_0^2 \varepsilon\kappa + \frac{\kappa}{\mu_z}\partial_y^2] \qquad (24c)$$

$$\hat{L}''_{22} = \frac{\mu}{\mu_z}\partial_x^2 + \partial_y^2 + k_0^2 \varepsilon\mu - j\frac{\kappa}{\mu_z}\partial_x\partial_y \qquad (24d)$$

The eq (22) is an equation with eigenvalues.

Let us suppose a rectangular guide charged by longitudinally magnetized ferrite as it is presented by the Fig.1. The decomposition of the field $\vec{E}_t$ on a complete base allows to obtain an eigenvalues system. The expressions of the transverse fields, verifying the limit conditions, can be expressed as follows:

$$E_x = \sum_{m,n=0}^{\infty} E_{x,mn} \cos\frac{m\pi}{a}x.\sin\frac{n\pi}{b}y = \sum_{m,n=0}^{\infty} E_x^{mn} \qquad (25)$$

$$E_y = \sum_{m,n=0}^{\infty} E_{y,mn} \sin\frac{m\pi}{a}x.\cos\frac{n\pi}{b}y = \sum_{m,n=0}^{\infty} E_y^{mn} \qquad (26)$$

We note: $f_m^x = \sin(\frac{m\pi}{a}x)$; $g_m^x = \cos(\frac{m\pi}{a}x)$

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$$f_n^y = \sin(\frac{n\pi}{b}y) \quad ; \quad g_n^y = \cos(\frac{n\pi}{b}y) \quad (26a)$$

The equations (23) and (24) can be expressed as follows:

$$\sum_{mn} \hat{L}_{11}'' E_{x,mn} g_m^x f_n^y + \hat{L}_{12}'' E_{y,mn} f_m^x g_n^y = k^2 \sum_{mn} E_{x,mn} f_n^y g_m^x \quad (27)$$

$$\sum_{mn} \hat{L}_{21}'' E_{x,mn} g_m^x f_n^y + \hat{L}_{22}'' E_{y,mn} f_m^x g_n^y = k^2 \sum_{mn} E_{y,mn} f_m^x g_n^y \quad (28)$$

Applying the Galerkin method

We have: $0 \le x \le a \;\; ; 0 \le y \le b$

We can choose the test functions as follows:

$$f_{m'}^x = \sin(\frac{m'\pi}{a}x) \quad ; \quad g_{m'}^x = \cos(\frac{m'\pi}{a}x)$$
$$f_{n'}^y = \sin(\frac{n'\pi}{b}y) \quad ; \quad g_{n'}^y = \cos(\frac{n'\pi}{b}y) \quad (28a)$$

The scalar product of $f_{n'}^y g_{n'}^x$ with the equation (27) and $f_m^n \; g_n^y$ with equation (28), and by integrating, we obtain:

$$\sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} [\hat{L}_{11}'' E_{x,mn} g_{m'}^x g_m^x f_{n'}^y f_n^y + \hat{L}_{12}'' E_{y,mn} g_{m'}^x f_m^x f_{n'}^y g_n^y]\, dxdy = k^2 \sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} E_{x,mn} g_{m'}^x g_m^x f_{n'}^y f_n^y .dxdy \quad (29)$$

$$\sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} [\hat{L}_{21}'' E_{x,mn} f_{m'}^x g_m^x g_{n'}^y f_n^y + \hat{L}_{22}'' E_{y,mn} f_{m'}^x f_m^x g_{n'}^y g_n^y]\, dxdy = k^2 \sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} E_{y,mn} g_{n'}^y g_n^y f_{m'}^x f_m^x .dxdy \quad (30)$$

Which can be written in the following matrix form

$$\sum_{m',n',m,n} \begin{bmatrix} H_{11\,mn}^{m'n'} & H_{12\,mn}^{m'n'} \\ H_{21\,mn}^{m'n'} & H_{22\,mn}^{m'n'} \end{bmatrix} \cdot \begin{bmatrix} E_{x,mn} \\ E_{y,mn} \end{bmatrix} = k^2 \sum_{m',n',m,n} \begin{bmatrix} G_{11\,mn}^{m'n'} & 0 \\ 0 & G_{22\,mn}^{m'n'} \end{bmatrix} \cdot \begin{bmatrix} E_{x,mn} \\ E_{y,mn} \end{bmatrix} \quad (31)$$

By normalising f_m^n, f_n^y, g_m^n, g_n^y we can easily find that

$$G_{11\,mn}^{m'n'} = G_{22\,mn}^{m'n'} = 1 \quad (31a)$$

We note : $H = \begin{bmatrix} H_{11\,mn}^{m'n'} & H_{12\,mn}^{m'n'} \\ H_{21\,mn}^{m'n'} & H_{22\,mn}^{m'n'} \end{bmatrix}.$ (31b)

The system (26) can be expressed as follows:

$$\boxed{H \cdot E_T = k^2 \cdot I \cdot E_T} \quad (32)$$

With I is the identity matrix. H is a square Matrix of $2N$ order with $N = m.n$: number of modes ; m and n are natural integers verifying (m,n) $\neq$ (0,0)

The eigenvalues and the eigenvectors of H are the propagation constants and the development coefficient of the real guide fields respectively.

IV. SIMULATIONS RESULTS

Table 1: Guide parameters of Fig.1.

Guide parameters	a mm	b mm	ε_f	$4\pi M_S$ Gauss
	22,86	10,16	14,5	600

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The simulations have been executed on a laptop computer under the platform Matlab. We obtain the propagation constant curves.

For a magnetised ferrite, $\left(\frac{k_z}{k_0}\right)^2$ becomes complex. We have plotted, on the one hand, the positive real modes (propagation modes) and the negative real modes (evanescent modes), see figures 2, 4 and 6, and, on the other hand, the complex modes (in this case, $\left(\frac{k_z}{k_0}\right)^2$ is imaginary), see figures 3, 5 and 7.

A. In the case where : $\frac{4\pi M}{4\pi M_s} = 0{,}3$

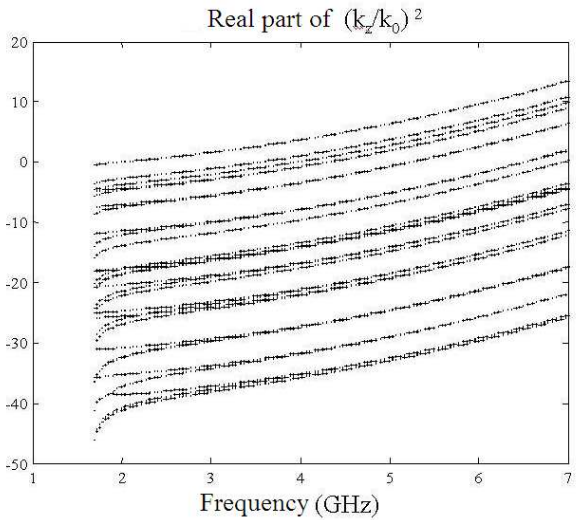

Fig.2. Dispersion curve : real of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

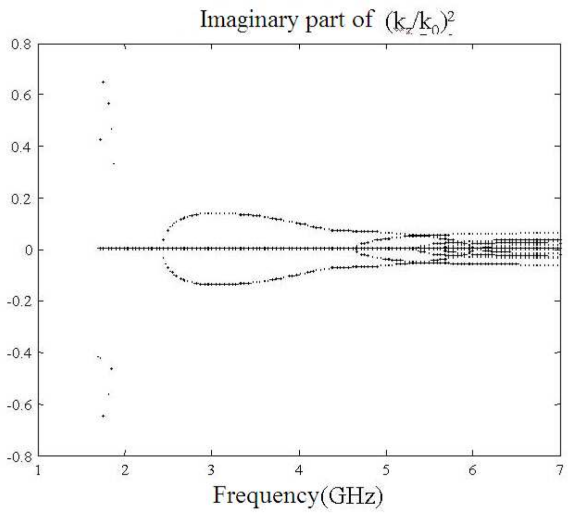

Fig.3 : dispersion curve : imaginary of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

B. The case where : $\frac{4\pi M}{4\pi M_s} = 0{,}5$

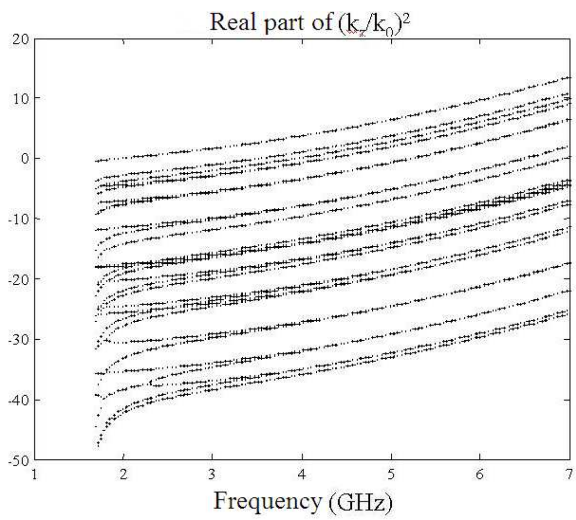

Fig.4 : : dispersion curve : real of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

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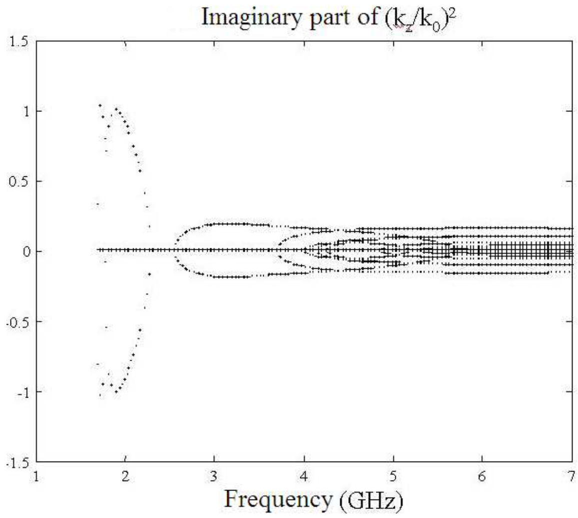

Fig.5: dispersion curve : imaginary of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

C. Details of complex modes for $\frac{4\pi M}{4\pi M_s} = 0,5$

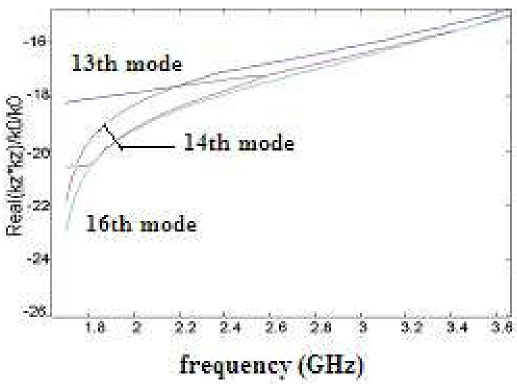

Fig.6 : dispersion curve : real of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

For a waveguide of longitudinally magnetized ferrite we obtain complex modes. The number of modes in this ferrite is larger than that in a dielectric guide (non magnetised ferrite).
This is due to the complex modes. Besides, for a given frequency, the propagation constant increases when the magnetization M increases.

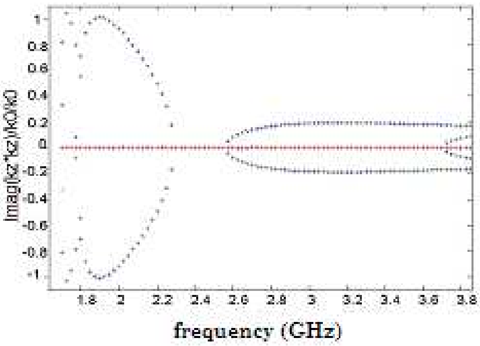

Fig.7: dispersion curve : imaginary of $\left(\frac{k_z}{k_0}\right)^2$ as a function of the frequency

V. CONCLUSION

The simulation results show clearly, by applying the transverse operator method, the existence of volume waves in a rectangular wave-guide totally full of longitudinally magnetized ferrite. With this formalism, we have presented a rigorous study of the propagation in anisotropic media. Moreover, we verify the existence of complex mode in this type of structures. This result is in a good agreement with that published by A.S.OMAR and al. [4] and Railton [5] (complex modes in anisotropic and inhomogeneous structures)

REFERENCES

[1] J. J. GREEN & F. SANDY, “Microwave characterization of partially magnetized ferrites,” *IEEE Trans-MTT* , vol.22, pp.641-645, June 1974.

[2] E. SCHLOEMANN, “Microwave behavior of partially magnetized ferrites,” *J. Appl. Phys.*, vol.1, p.1204, June 1970.

[3] J. W. TAO, R. ANDRIAMANJATO & H. BAUDRAND, "General waveguide problems studies by transverse operator method, " *IEEE Trans-Magnetics*, Vol.31, Issue 3, pp.1626 – 1629, May 1995

[4] E. H A.S.OMAR, K.F.SCHUNEMANN, "Complex and backward- wave modes in inhomogeneously and anisotropically filled waveguides," *IEEE Trans-MTT*, vol.35, pp.268-275, Mars 1987.

[5] C. J. RAILTON and T. ROZZI, "Complex Modes in Boxed Microstrip" *IEEE Trans MTT*, Vol 36, N° 5, May 1988, pp 865-874.